PHYC/ECE 463 Advanced Optics I

Fall 2007

Homework #10, Due Wednesday Nov. 14

1- K&F 5.31

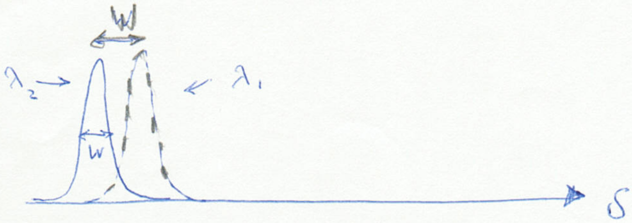
2- K&F 5.32 (Use matrix method and possibly a mathematical software)

3- K&F 5.35

Note: the equation for L matrix given in this problem is wrong (as stated in the class)! Correct it before proceeding with the problem. In addition, (b) Calculate the new W (full-width-half-maximum (FWHM) of the transmittance) in terms of F and T_{max}. (c) Plot the transmission versus δ for β'=0.12, 0.1, 0, and -0.15. (c) Explain the physical meaning and consequences of the condition for which T_{max} becomes ∞.)

4- K&F 5.37

5-31



$$W = \frac{4}{\sqrt{F}} = \frac{2(1-R_1)}{\sqrt{R_1}}$$

$$R = \frac{\lambda}{\Delta\lambda} = m\mathcal{F} = m\,\frac{\pi\sqrt{F}}{2} = m\,\frac{\pi\sqrt{R_1}}{1-R_1}$$

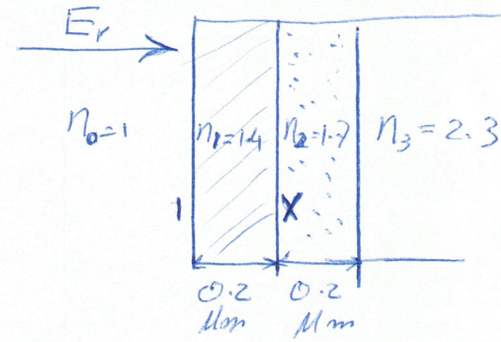
$$\boxed{m = \frac{(1-R_1)\,\lambda}{\Delta\lambda\,\pi\sqrt{R_1}}}$$

$$\left.\begin{aligned}\lambda_1 &= 599.4 \text{ nm}\\ \lambda_2 &= 600.0 \text{ nm}\end{aligned}\right\} \quad \begin{aligned}\Delta\lambda &= \lambda_2 - \lambda_1 = 0.6 \text{ nm}\\ \lambda &\approx 600 \text{ nm}\end{aligned}$$

$$R_1 = 0.85 \Rightarrow m = \frac{600 \times 0.15}{0.6 \times \pi \times \sqrt{0.85}} = 51.8$$

Therefore, the two wavelengths would be resolved @ $m \geq 52$

5.32: Using Mathcad, we can calculate R and T for a range of wavelength (e.g. $0.4 \to 0.7\ \mu m$)



$i := \sqrt{-1}$ $\quad \lambda := 0.4, .41 .. .7$

$n := (1\ \ 1.4\ \ 1.7\ \ 2.3)$

$d := (0\ \ .2\ \ .2\ \ 0)$

$\boxed{\beta(j,\lambda) := \frac{2\cdot\pi}{\lambda}\cdot n_{0,j}\cdot d_{0,j}}$ $\quad \boxed{\rho(j) := \frac{n_{0,j+1} - n_{0,j}}{n_{0,j+1} + n_{0,j}}}$ $\quad \boxed{\tau(j) := \frac{2\cdot n_{0,j}}{n_{0,j+1} + n_{0,j}}}$

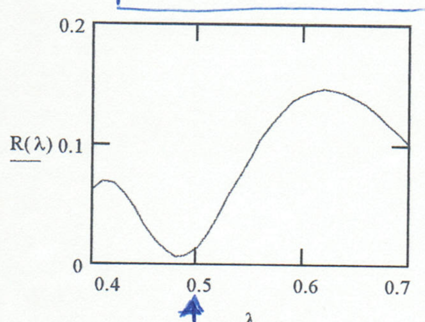
$H(j) := \begin{pmatrix} 1 & \rho(j) \\ \rho(j) & 1 \end{pmatrix}\cdot\frac{1}{\tau(j)}$ $\quad L(j,\lambda) := \begin{pmatrix} \exp(-i\cdot\beta(j,\lambda)) & 0 \\ 0 & \exp(i\cdot\beta(j,\lambda)) \end{pmatrix}$

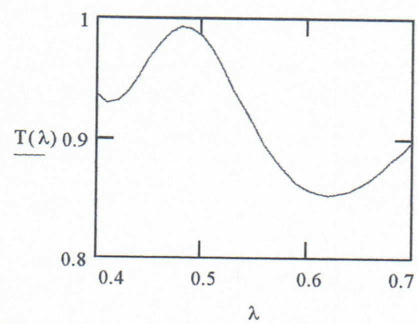
* See Note

$\boxed{S(\lambda) := H(0)\cdot L(1,\lambda)\cdot H(1)\cdot L(2,\lambda)\cdot H(2)}$ $\quad N := \mathrm{cols}(n) - 1$ $\quad \boxed{r(\lambda = 500\,nm) = -0.091 + 0.087\,i}$

$t(\lambda) := \frac{1}{S(\lambda)_{1,1}}$ $\quad r(\lambda) := \left(\frac{S(\lambda)_{0,1}}{S(\lambda)_{1,1}}\right)$ $\quad T(\lambda) := t(\lambda)\cdot\overline{t(\lambda)}\cdot\frac{(\mathrm{Re}(n))_{0,N}}{(\mathrm{Re}(n))_{0,0}}$ $\quad R(\lambda) := r(\lambda)\cdot\overline{r(\lambda)}$

$\Rightarrow$ **R=0.016 at 500 nm** $\Leftarrow$





Note $R + T = 1$ since there is no loss

from 1 to X $\quad \lambda := .5$

$\Rightarrow S_{13} := H(0)\cdot L(1,\lambda)\cdot H(1)$, $\quad S_{13} = \begin{pmatrix} -1.255 + 0.481i & -0.325 - 0.034i \\ -0.325 + 0.034i & -1.255 - 0.481i \end{pmatrix}$

$\begin{pmatrix} E_l \\ E_r \end{pmatrix}_{at\,X} = S_{13}^{-1}\begin{pmatrix} E_{l1}' \\ E_{r1}' \end{pmatrix} = S_{13}^{-1}\begin{pmatrix} r E_{r1}' \\ E_{r1}' \end{pmatrix}$

$\Rightarrow \begin{pmatrix} E_l \\ E_r \end{pmatrix} = S_{13}^{-1}\cdot\begin{pmatrix} r \\ 1 \end{pmatrix}\times E_{r1}' = \begin{pmatrix} -0.099 - i0.058 \\ -0.73 + i0.265 \end{pmatrix} E_{r1}'$

$E(at\,x) = (E_l + E_r)_{at\,X} = (-0.829 + .207\,i)E_{r1}' = 0.85 \times e^{i2.9} E_{r1}'$

* Note: In Mathcad notation, matrices are defined as:

$$S = \begin{pmatrix} S_{00} & S_{01} \\ S_{10} & S_{11} \end{pmatrix} \quad (2\times 2 \text{ example})$$

#3

5.35 Unfortunately, in K&F, there is a misprint!

L matrix, when there is gain, should be in the form of

$$L = \begin{pmatrix} e^{-i\beta+\beta'} & 0 \\ 0 & e^{+i\beta-\beta'} \end{pmatrix} \quad \text{when} \quad \begin{array}{l} \beta' > 0 \text{ gain} \\ \beta' < 0 \text{ loss.} \end{array}$$

You can derive this from Eqns $\underline{5.81}$ in K&F (pp 298)

Therefore, only by replacing $\underline{\beta}$ ~~with~~ $\beta - i\beta'$ in the transmission coeff. τ (or ρ), we obtain:

$$\tau = \frac{\tau_{12}\tau_{21} e^{-i\beta+\beta'}}{1-\rho_{12}^2 e^{-i2(\beta-i\beta')}}$$

$$T = |\tau|^2 = \frac{|\tau_{12}\tau_{21}|^2 e^{+2\beta'}}{\left|1-\rho_{12}^2 e^{+2\beta'} e^{-i2\beta}\right|^2}$$

let $\left.\begin{array}{l}\rho_{12}^2 = R_1 \\ \tau_{12}\tau_{21} = 1-R_1\end{array}\right\}$ lossless interfaces

$$T = \frac{(1-R_1)^2 e^{2\beta'}}{1+R_1^2 e^{4\beta'} - 2R_1 e^{2\beta'}\cos\delta} = \frac{\dfrac{(1-R_1)^2 e^{2\beta'}}{(1-R_1 e^{2\beta'})^2}}{1+\dfrac{4R_1 e^{2\beta'}}{(1-R_1 e^{2\beta'})^2}\sin^2\delta/2}$$

$$T = \frac{\dfrac{(1-R)^2 e^{2\beta'}}{(1-R_1 e^{2\beta'})^2}}{1+\dfrac{4R_1}{(1-R_1)^2}\times\dfrac{(1-R)^2 e^{2\beta'}}{(1-R_1 e^{2\beta'})^2}\sin^2\dfrac{\delta}{2}} = \frac{T_{max}}{1+F\times T_{max}\times\sin^2\dfrac{\delta}{2}}$$

$$F = \frac{4R_1}{(1-R_1)^2}, \quad T_{max} = \frac{(1-R)^2 e^{2\beta'}}{(1-R_1 e^{2\beta'})^2}$$

⊚ Plot $\underline{T}$ vs. $\underline{\delta}$ for $\beta' = \underbrace{0.1, 0.12}_{\text{gain}}, \underset{\uparrow \text{ Passive}}{0}, \underset{\uparrow \text{ loss}}{-.15}$

$\beta1 := .1 \quad \beta2 := .12 \quad \beta3 := .0 \quad \beta4 := -.15$

$\delta := 3, 3.01 .. 15$

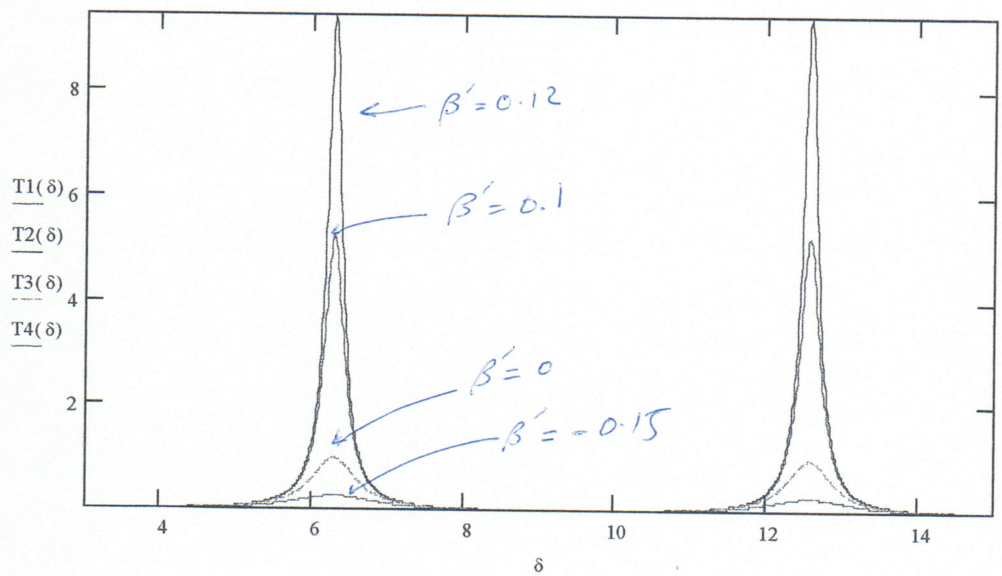
$R := .7 \qquad F := \dfrac{4 \cdot R}{(1-R)^2} \qquad \mathrm{Tmax}(\beta') := \dfrac{(1-R)^2 \cdot \exp(2 \cdot \beta')}{(1 - R \cdot \exp(2 \cdot \beta'))^2}$

Here $\boxed{F = 31.1}$

$T(\delta, \beta') := \dfrac{\mathrm{Tmax}(\beta')}{1 + F \cdot \mathrm{Tmax}(\beta') \cdot \sin\left(\dfrac{\delta}{2}\right)^2}$

$T1(\delta) := T(\delta, \beta1) \quad T2(\delta) := T(\delta, \beta2)$

$T3(\delta) := T(\delta, \beta3) \quad T4(\delta) := T(\delta, \beta4)$



Note: ① $T_{max} \to \infty$ when $R_1 e^{2\beta'} = 1$

This is the condition for laser action threshold (oscillation)

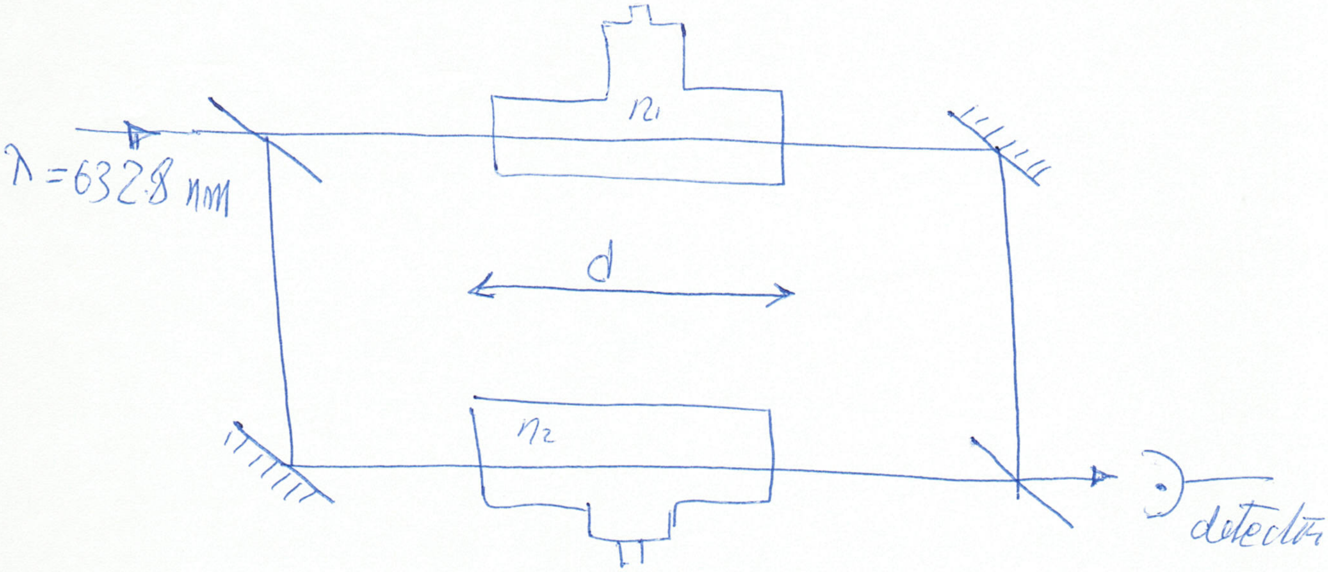
② with $\beta' > 0$ (but bellow laser threshold, i.e. $\beta' < \frac{1}{2} \ln \frac{1}{R_1}$) the W_{FWHM} decreases. Conversely, it increases if we have loss ($\beta' < 0$).

⊚ We can derive W_{FWHM} when $|\delta - 2m\pi| \ll 1$

$$\boxed{W_{FWHM} \approx \frac{4}{\sqrt{F}} \times \frac{1}{\sqrt{T_{max}}}}$$

This is called * Gain Narrowing (for $\beta' > 0$)

#3 (5.37)



$d = 10$ cm

$n_1 = 1$

$n_2 = 1 + Ap$

$$\delta = \frac{2\pi}{\lambda} d\,(n_1 - n_2) = \frac{2\pi}{\lambda} d\,(1 - 1 - Ap) = -\frac{2\pi d}{\lambda} Ap$$

$$S(\delta) = S_{max} \cos^2 \frac{\delta}{2} \qquad \delta = 2m\pi \ \text{(bright fringes)}$$

$$\Rightarrow \frac{d}{\lambda} Ap = m \qquad \Delta m = \frac{d}{\lambda} A \Delta p$$

$$A = \frac{\Delta m}{\Delta p} \times \frac{\lambda}{d} = \frac{100}{1\ \text{atm}} \times \frac{632.8 \times 10^{-9}}{0.1} = 632.8 \times 10^{-6}$$

$$= 0.6328 \times 10^{-3}$$

per atm.